

separation radius of the vortices; c – cooled flow; h – heated flow; s – supplemented flow; cr – critical cross section; * – deceleration parameter; z – axial component; φ – circular component; r – radial component; st – static parameter; meas – measurement; ' – pulsed component.

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CALCULATION OF THE CHARACTERISTICS OF PARTICLES IN INHOMOGENEOUS TURBULENT STREAMS

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The influence of inertia, averaged phase slip, and inhomogeneity of the turbulent carrier stream on the parameters of the disperse phase in the flow of a gas suspension in a round pipe is investigated on the basis of a closed system of equations for the first and second moments of particle velocity pulsations.

The averaged characteristics of a disperse phase (particle concentration, average velocity) depend essentially on the intensity of the pulsating motion of the particles. The pulsation energy of the disperse phase is determined by three main factors. The first is the particle inertia, equal to the ratio of the time of dynamic relaxation of particles to the integrated macroscopic time scale of turbulence of the carrier stream. As the time of dynamic relaxation of the particles decreases, the pulsation energy of the discrete phase approaches that of the carrier phase. An increase in the time of dynamic relaxation of the particles in comparison with the lifetime of energy-carrying vortices decreases the degree of entrainment of particles in the pulsating motion. The influence of particle inertia on the pulsation characteristics of a disperse admixture in a homogeneous turbulent stream has been well studied, in [1, 2], for example.

Second, in particle flow under the conditions of averaged velocity slip, an effect of "crossing of trajectories" of particles and turbulent floats arises. As a result of the continuous renewal of turbulent floats crossing a particle trajectory, the autocorrelation function of pulsations of gas velocity along a particle trajectory decays faster than the original autocorrelation function of gas velocity pulsations, leading to a decrease in the intensity of pulsating motion of the discrete admixture and to a decrease in the turbulent diffusion coefficient of the particles. In the case of isotropic turbulence, the action of the averaged velocity slip of the phases on the pulsation characteristics of the particles has been investigated both experimentally and theoretically in [3-8].

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The third factor that determines the level and character of the spatial variation of pulsation energy of the discrete phase in streams with velocity shear is the nonuniformity of the field of turbulent velocity pulsations of the carrier phase, especially near a confining solid surface. Nonlocality effects, when the pulsation characteristics of particles at a given point of space depend integrally on the characteristics of the liquid phase, are important in this case. The nonlocal character of the pulsation energy of the disperse admixture is due to the correlation between particle trajectories leading to the given point of space, and it increases with increasing particle inertia. Nonzero velocity and pulsation energy of the disperse phase at the surface are observed as a result, leading to the development of intense averaged phase slip and the deposition of particles onto the channel walls. The effect of averaged phase slip has been investigated experimentally (see, e.g., the review in [9]), by direct stochastic simulation of individual particle trajectories [10-12], and by solving the system of equations for the first and second moments [13]. A fairly complete review of calculation methods and experimental data on particle deposition in turbulent flow in channels is given in [14]. Of more recent papers, we note [15-19].

From the foregoing, it follows that simultaneous allowance for all factors is necessary for a correct description of the flow of a disperse phase in an inhomogeneous turbulent stream. In the present paper we calculate the distributions of concentration and deposition rate of an admixture, the averaged phase slip, and the intensity of pulsating motion of particles in pipes on the basis of a closed system of equations for the averaged velocity and concentration, as well as the intensity of transverse velocity pulsations of the disperse admixture, with boundary conditions allowing for the character of the interaction of particles with the channel walls [13]. The influence of the "trajectory crossing" effect on the coefficients of turbulent transport of momentum and mass of the disperse phase is described in accordance with [20].

1. The equations and boundary conditions for calculating the intensity of transverse pulsations of particles and the averaged velocity of the disperse phase in cylindrical coordinates have the form [13]

$$\begin{aligned} \frac{1}{(1-\bar{y})} \frac{d}{d\bar{y}} \left[(1-\bar{y}) D_+ \frac{d\sigma_+}{d\bar{y}} \right] - R_+ V_{y+} \frac{d\sigma_+}{d\bar{y}} - \\ - \frac{2R_+^2}{\tau_+} \sigma_+ = -2 \frac{R_+^2}{\tau_+} f e_+; \quad \frac{d\sigma_+}{d\bar{y}} = 0, \quad \bar{y} = 1, \end{aligned} \quad (1)$$

$$D_+ \frac{d\sigma_+}{d\bar{y}} - R_+ \left[V_{y+} + 2 \left(\frac{2}{\pi} \sigma_+ \right)^{1/2} \frac{1 - \kappa_2^3}{1 + \kappa_2^3} \right] \sigma_+ = 0, \quad \bar{y} = 0, \quad (2)$$

$$\begin{aligned} \frac{1}{(1-\bar{y})} \frac{d}{d\bar{y}} \left[(1-\bar{y}) \frac{\tau_+ \sigma_+}{2} \frac{d\bar{V}_x}{d\bar{y}} \right] - R_+ V_{y+} \frac{d\bar{V}_x}{d\bar{y}} - \\ - \frac{R_+^2}{\tau_+} \bar{V}_x = - \frac{R_+^2}{\tau_+} \bar{U}_x, \quad \frac{d\bar{V}_x}{d\bar{y}} = 0, \quad \bar{y} = 1, \end{aligned} \quad (3)$$

$$\frac{\tau_+ \sigma_+}{2} \frac{d\bar{V}_x}{d\bar{y}} - R_+ \left[V_{y+} + \left(\frac{2}{\pi} \sigma_+ \right)^{1/2} \frac{1 - \kappa_1 \kappa_2}{1 + \kappa_1 \kappa_2} \right] \bar{V}_x = 0, \quad \bar{y} = 0, \quad (4)$$

where σ_+ and e_+ are the second moments of transverse velocity pulsations of the disperse and continuous phases in dynamic variables; V_{y+} is the transverse velocity of the disperse phase in dynamic variables; \bar{V}_x is the longitudinal velocity of the disperse phase, normalized to the average-mass velocity of the carrier phase; κ_1 and κ_2 are the coefficients of restitution of particle momentum in collisions with the channel walls in the longitudinal and transverse directions. The turbulent diffusion coefficient D_+ of the particles in the dynamic variables is calculated from the equation

$$D_+ = \tau_+ (\sigma_+ + g e_+). \quad (5)$$

In calculating the time τ_+ of dynamic relaxation of particles in the universal variables, we allow for the velocity of streamline flow due to the averaged phase slip.

The particle concentration and the transverse velocity of the disperse phase are calculated from the equation

$$-\bar{c}V_{y+} = D_+ \frac{d\bar{c}}{dy} - R_+(V_{mig+} + U_{y+})\bar{c} = R_+(1 - \bar{y})J_+, \quad (6)$$

$$J_+ = \left(\frac{2}{\pi} \sigma_+ \right)^{1/2} \frac{1 - \kappa_2}{1 + \kappa_2} \bar{c}, \quad \bar{y} = 0, \quad (7)$$

$$V_{mig+} = -\frac{\tau_+}{R_+} \frac{d\sigma_+}{dy}, \quad \bar{c} = c/c_m, \quad c_m = 2 \int_0^1 d\bar{y} (1 - \bar{y}) c(\bar{y}), \quad (8)$$

where V_{mig+} is the migration velocity of the particles, due to the spatial nonuniformity of turbulent energy of the discrete phase, and $J_+ = V_y/u_+$ is the dimensionless rate of deposition of particles of the admixture onto the channel walls.

2. The influence of the averaged velocity of phase slip on the degree of entrainment of particles into the pulsating motion of the carrier phase is calculated on the basis of allowance for nonlocal effects due to the different degrees of particle entrainment in the small-scale pulsating motion that forms an energy-carrying float [20]. The source term fe_+ in Eq. (1) for the intensity of transverse particle pulsations and the term ge_+ in Eq. (5) for the turbulent diffusion coefficient of the particles are written in the form

$$fe_{ii}(\mathbf{x}, t) = \frac{1}{\tau} \int dy \int_0^\infty ds \exp\left(-\frac{s}{\tau}\right) R_{ii}(\mathbf{x}; \mathbf{y}, s) G(\mathbf{y}, s), \quad (9)$$

$$ge_{ii}(\mathbf{x}, t) = \frac{1}{\tau} \int dy \int_0^\infty ds \left[1 - \exp\left(-\frac{s}{\tau}\right) \right] R_{ii}(\mathbf{x}; \mathbf{y}, s) G(\mathbf{y}, s), \quad (10)$$

$$R_{ii}(\mathbf{x}; \mathbf{y}, s) = \langle u_i(\mathbf{x}, t) u_i(\mathbf{x} + \mathbf{y}, t + s) \rangle, \quad e_{ii}(\mathbf{x}, t) = \langle u_i^2(\mathbf{x}, t) \rangle,$$

$$G(\mathbf{y}, s) = \prod_{i=1}^3 (2\pi l_i^2)^{-1/2} \exp\left[-\frac{(y_i - W_i s)^2}{2l_i^2}\right],$$

$$l_i^2 = \tau^2 f_0^2 (\sigma_{ii} + e_{ii}), \quad f_0 = 1 - \exp(-T_p/\tau), \quad W_i = |U_i - V_i|, \quad (11)$$

where $R_{ii}(\mathbf{x}; \mathbf{y}, s)$ is the correlation of velocity pulsations of the carrier phase in the i -th direction, calculated for a particle trajectory; $G(\mathbf{y}; s)$ is the probability density for the transport of particles over a distance \mathbf{y} in a time s ; l_i^2 is the square of the characteristic displacement of a particle within an energy-carrying float in a time $s \approx T_p$ (T_p is the characteristic time of interaction of a particle with a turbulent float). Since the Lagrangian and Eulerian scales of velocity pulsations of the carrier phase differ considerably in magnitude [21, 22], in determining the degree of entrainment of the disperse admixture in the turbulent motion one must allow for the transformation from the Lagrangian characteristics of pulsations of the carrier phase along the trajectory of a low-inertia particle to the Eulerian characteristics along the trajectory of a high-inertia particle if the averaged velocity slip of the phases is significant [23]. As the quantitative criterion for this transformation, we take the ratio $\alpha = \langle y_p^2 \rangle^{1/2}/L_L$, where

$$\langle y_p^2 \rangle = \int_{-\infty}^{\infty} dy y^2 G(y, T_L) = W^2 T_L^2 + l^2.$$

The characteristic scales of decay of the correlation of gas velocity pulsations along a particle trajectory are calculated from the equation

$$\beta_p = \frac{\alpha + \beta}{\alpha + 1}, \quad \beta_p = \frac{T_p}{T_E} = \frac{L_p}{L_E}, \quad \beta = \frac{T_L}{T_E} = \frac{L_L}{L_E},$$

where L_E and T_E are the Eulerian spatial and temporal macroscopic scales of gas velocity pulsations, measured in the coordinate system moving with the averaged stream velocity. Without allowance for the influence of the inertial of the admixture on the characteristic time of interaction of particles with a turbulent float ($l_i^2 = 0$), from (9)-(11) we have

$$fe(y) = \frac{1}{\tau} \int_0^\infty ds \exp\left(-\frac{s}{\tau}\right) R(y; Ws, s), \quad (12)$$

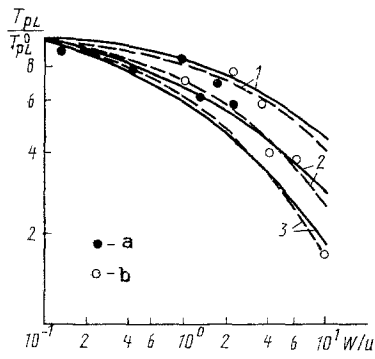


Fig. 1

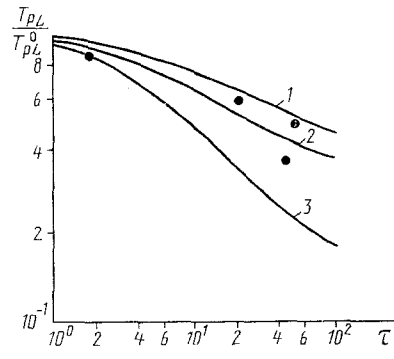


Fig. 2

Fig. 1. Influence of the phase slip velocity on the Lagrangian time scale of particle velocity pulsations: solid curves) $d_p = 5 \mu\text{m}$; dashed curves) $57 \mu\text{m}$; curves) calculation; points) experiment [4]; a) $d_p = 5 \mu\text{m}$; b) $57 \mu\text{m}$; 1) $x/M = 30$; 2) 45 ; 3) 90 .

Fig. 2. Lagrangian time scale of particle velocity pulsations as a function of particle dynamic relaxation time in the case of gravitational settling: curves) calculation; points) experiment [3]; 1) $x/M = 41$; 2) 73 ; 3) 171 ; τ , μsec .

$$ge(y) = \frac{1}{\tau} \int_0^{\infty} ds \left[1 - \exp\left(-\frac{s}{\tau}\right) \right] R(y; Ws, s). \quad (13)$$

In the case of the approximation $R(y; z, s) = e(y)\varphi(z, s)$, $\varphi(z, s) = \exp(-|z|/L_p - |s|/T_p)$, we obtain

$$\begin{aligned} f &= [1 + \Omega/\beta_p(1 + \alpha)]^{-1}, \quad \alpha = W/u, \\ g &= \beta_p/\Omega(1 + \alpha)^{-1} - f, \quad \Omega = \tau/T_E, \quad u = e^{1/2}. \end{aligned} \quad (14)$$

3. Let us illustrate the influence of the averaged velocity slip of the phases on the pulsation characteristics of particles in the case of uniform isotropic turbulence. For a comparison with existing experimental data [3, 4], we calculate the Lagrangian time scale of velocity pulsations of the discrete phase:

$$\begin{aligned} T_{pL} &= \int dy \int_0^{\infty} ds \varphi(y, s) G(y, s) \times \\ &\times \left[\frac{1}{\tau} \int dy \int_0^{\infty} ds \exp\left(-\frac{s}{\tau}\right) \varphi(y, s) G(y, s) \right]^{-1}. \end{aligned}$$

In the case $l = 0$ and the previously chosen approximation of the function φ , we have

$$\frac{T_{pL}}{T_E} = \beta_p \frac{1 + \Omega/\beta_p(1 + \alpha)}{(1 + \alpha)}.$$

In Figs. 1 and 2 we compare experimental and calculated data ($\beta = 0.5$ [22]) on the influence of the relative velocity on the Lagrangian time scale of particle velocity pulsations for a low-inertia admixture ($\Omega < 1$). In the case of Fig. 1, the phase slip was caused by an electric field acting on charged particles; in the case shown in Fig. 2, the velocity mismatch was due to gravitational settling of particles.

We used the system of equations (1)-(8) and Eqs. (14) to calculate particle flow in channels with absorbing and reflecting walls. The system (1)-(4) was numerically integrated by the sweep method with interactions on a nonuniform grid with 70 nodes (the relative calculation error was less than $5 \cdot 10^{-3}$). We solved Eq. (5) for the particle concentration by the predictor-corrector method (a three-step algorithm [24]). For low-inertia particles ($\tau_+ \leq 10$) the grid was bunched near the boundaries so that there were at least six grid nodes within a distance $\sim \tau_+$ from the wall, in which case the boundary

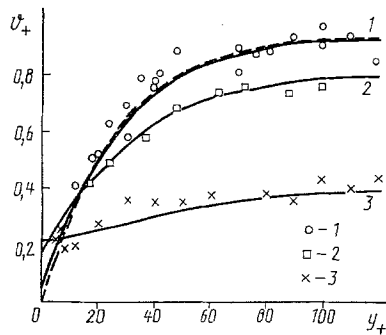


Fig. 3. Distribution of the transverse component of particle pulsation velocity near the channel wall: points) data of [18]; curves) calculation; 1) $\tau_+ = 1$; 2) 15; 3) 100.

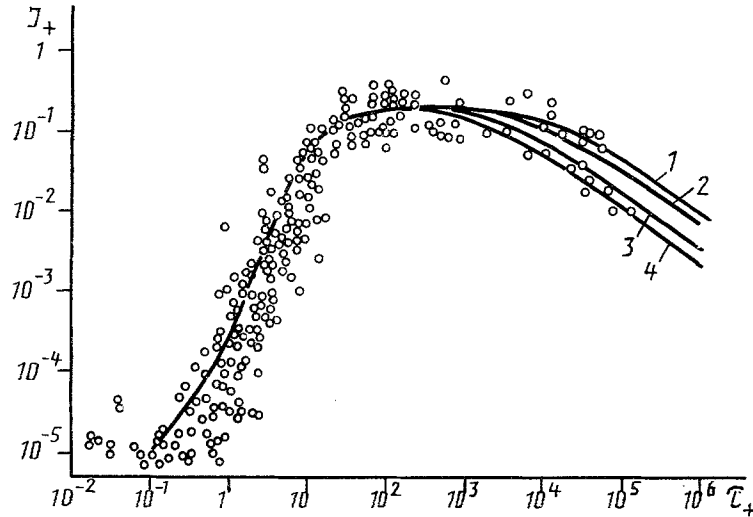


Fig. 4. Influence of particle inertia on the deposition rate of the disperse admixture: points) data of [14, 18, 28]; curves) calculation: 1) $R_+ = 5 \cdot 10^3$; 2) $3 \cdot 10^3$; 3) $1 \cdot 10^3$; 4) $6 \cdot 10^2$.

conditions in the interior of the stream were chosen from the condition of asymptotic approach of the characteristics of the disperse phase to the corresponding characteristics of the carrier stream. The distribution of the intensity of transverse velocity pulsations of the carrier stream was specified in the form [25] $e_+(y_+) = b[1 - \exp(-y_+/A_+)]^2$, where $A_+ = 30$ and $b \approx 1$. The averaged velocity profile of the carrier stream was chosen in the form of the Reichardt approximation [26]. The Eulerian macroscopic time scale of turbulent pulsations across the channel was taken to be 10 for $y_+ \leq 5$, and for larger distances from the wall it was calculated from the equation [27]

$$T_{E+} = \gamma L_{E+} / e_+^{1/2}, \quad \gamma = 1.16,$$

where the scale of the turbulence was determined by Nikuradse's method.

The distribution of the transverse pulsation velocity $v_+ = \sigma_+^{1/2}$ of particles in the case of the flow of drops ($\kappa_2 = 0$) in a round pipe ($R_+ = 10^3$) is shown in Fig. 3. It is seen that for high-inertia particles, the pulsation velocity of the admixture at the wall differs from zero. With increasing particle inertia, the turbulent energy of the particles varies little across the channel. The intensity of penetration of particles into the wall region of flow determines the rate of deposition of the disperse admixture onto the channel walls. Figure 4 illustrates the dependence of the deposition rate on the dimensionless time of dynamic relaxation of the particles, calculated in the Stokes approximation. The experimental data were borrowed from [14] and supplemented from [18, 28]. For $\tau_+ < 10^2$, a universal domain is observed in which the deposition intensity is determined by the behavior of particles in the region near the channel wall. For $\tau_+ > 10^3$, the deposition intensity depends on the degree of particle entrainment into pulsating motion in the stream core. The turbulent energy of the admixture then depends on the ratio τ_+/R_+ . For very high-inertia particles ($\tau_+ > 10^4$), a decrease in the deposition rate is observed because of the decrease

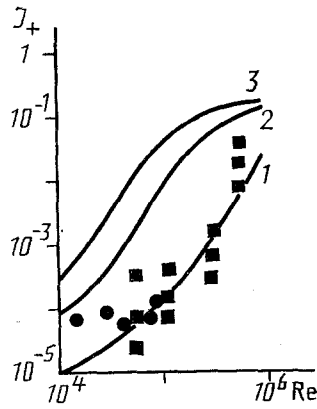


Fig. 5

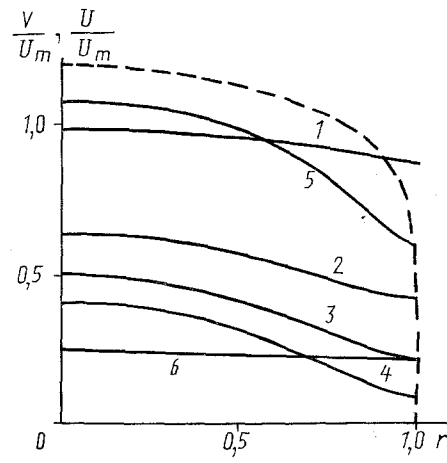


Fig. 6

Fig. 5. Influence of the stream Reynolds number on the particle deposition rate: points) data of [14]; curves) calculation: 1) $\tau_+^0 = 0.1$; 2) 1; 3) 5.

Fig. 6. Distribution of the averaged velocity of the disperse phase over a pipe cross section for different conditions of interaction of particles with the channel walls ($R_+ = 3 \cdot 10^3$): dashed line) \bar{U}_x ; 1) $\tau_+ = 10^5$, $\kappa_1 = 0.5$, $\kappa_2 = 0$; 2) $\tau_+ = 10^5$, $\kappa_1 = 0.8$, $\kappa_2 = 1$; 3) $\tau_+ = 10^5$, $\kappa_1 = 0.5$, $\kappa_2 = 1$; 4) $\tau_+ = 10^5$, $\kappa_1 = 0$, $\kappa_2 = 1$; 5) $\tau_+ = 10^4$, $\kappa_1 = 0.8$, $\kappa_2 = 1$; 6) $\tau_+ = 10^6$, $\kappa_1 = 0.8$, $\kappa_2 = 1$.

in turbulent energy of the admixture in the stream core. By varying the Reynolds number of the stream, one can control the intensity of particle deposition onto the walls (Fig. 5). A sharp increase in deposition rate with increasing Reynolds number occurs for particles with initially small $\tau_+^0 \approx 10^{-1}$. For an admixture of larger particles, the deposition depends less strongly on the Reynolds number.

In Fig. 6 we give the distribution of the averaged particle velocity as a function of the nature of the interaction with the channel walls. In the case of the flow of drops (an absolutely absorbing surface, $\kappa_2 = 0$), the average-mass velocities of the admixture and the carrier phase are similar. The discrepancy in axial velocities is caused by nonlocal effects due to particle inertia. Intense velocity slip between the phases develops in a channel with walls that restore particle momentum in the transverse direction ($\kappa_2 = 1$) and decrease the longitudinal momentum ($\kappa_1 < 1$). The averaged slip then increases with increasing ratio τ_+/R_+ and decreasing momentum restitution coefficient κ_1 .

The method given here for calculating the averaged and pulsation characteristics of the disperse phase in an inhomogeneous turbulent stream reflects the main relationships in that complicated class of two-phase flows.

NOTATION

σ , e , intensities of transverse velocity pulsations of particles and carrier gas; R , channel radius; τ , dynamic relaxation time of particles; V_x , U_x , averaged flow velocities of the disperse phase and the carrier stream; V_y , U_y , transverse flow velocities of the disperse phase and the gas; $\bar{y} = y/R$, relative transverse coordinate, measured from the channel wall; D , turbulent diffusion coefficient of the particles; c , c_m , particle concentration and its average-mass value; J , rate of deposition of particles onto the channel wall; W , modules of the relative velocity of particles and gas; T_E , L_E , Eulerian temporal and spatial macroscopic scales of gas velocity pulsations; T_L , L_L , Lagrangian temporal and spatial macroscopic scales of gas velocity pulsations; d_p , particle diameter; x/M , ratio of the downstream distance from the turbulizing grid to the cell size M ; T_{pL} , Lagrangian temporal macroscopic scale of particle velocity pulsations; $v = \sigma^{1/2}$, root-mean-square velocity of particles in the transverse direction; u_+ , dynamic stream velocity. Index: +, quantities measured in dynamic variables. An overbar denotes a variable normalized to its average-mass value.

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